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Solving Enneagonal Transportation Problem by Various Mechanism in Pythagorean Fuzzy Territory

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Abstract: *The Pythagorean Enneagonal Transportation Problem (PEGTP) is used to determine the quantity and need of goods transported from one source to various destinations. In this paper, the Enneagonal Pythagorean Fuzzy Numbers are used to find the Transportation Problem utilizing the proposed ranking method and Centroid Ranking Technique. By adopting the proposed ranking method under the score function method, the cost of transportation can be reduced. A numerical example is used to demonstrate the process.*

Keywords: Pythagorean Transportation problems, Enneagonal fuzzy numbers, ranking method, Centroid Ranking Technique, proposed ranking method, Initial Basic Feasible Solution, Optimal Solution.

Introduction

Practically speaking, businesses and industries must minimize costs for non-economic products that are essential to the survival of their companies. The primary focus of transportation models or problems is on the most efficient (best) way to move the goods produced at several factories or plants (referred to as supply origins) to numerous warehouses or customers (referred to as demand destinations). The target of a transportation problem is to satisfy the destination demands while operating within the restrictions of production availability at the minimum cost. A well-built accord to meet up the confront of how to provide the merchandise to the customers in more adept approaches is accomplished with the aid of transportation models Hitchcock in 1941 developed the essential transportation problem. Decision making field is one of the most significant field which enables to choose the best option among the feasible ones. Prof. Zadeh introduced the concept of fuzzy set theory to cope up with uncertainty and fuzziness of things in real life problems. Fuzzy set has a membership function is defined to assign the degree of membership of an element from the universe to a unit interval [0, 1]. Fuzzy set seems to be incomplete due to the omission of non-membership function and neglect of hesitation degree , Atanassov introduced the concept of Intuitionistic Fuzzy Set [IFS] in 1986, such that and Atanassov constructed the generalization of IFS which deals with the situation known as Intuitionistic Fuzzy Set of second type [IFSST]. A new theory encompassing philosophical belief which addresses nature and possibility of neutralities was introduced by Florentin Smarandache in 1995. A Neutrosophic Set (NS) considers truth membership (TM), inderminancy membership (IDM) and falsity membership (FM). Wang et. al in 2005, defined an illustration of neutrosophic set known as single valued neutrosophic set. A single valued neutrosophic number is a special type of neutrosophic set which plays with uncertain and vagueness data. In day-to-day life there be situations such that incorporate this situation a non-standard fuzzy subset was defined by Yager in 2013, known as Pythagorean Fuzzy set [PFS]. It satisfies the condition where is a Pythagorean fuzzy index. PFS consists uncertain and complex information more efficient than IFS, it is easily applied in solving decision making, science and engineering problems. Transportation Problems suitable the commodities to the customers in more proficient approaches. Intuitionistic Fuzzy Sets (ỈFSs) initiated by Atanassov who has tinted these ideas and delivered a tool. This Problem deals both membership function "μ" and non-membership function "ν" with hesitation range "π" such that $\mu + \nu \leq 1$ and $\mu + \nu + \pi = 1$. He fixed on Intuitionistic Fuzzy Sets Next Type (ỈFSNT) with the condition that the sum of the square of the membership and non-membership grade is less than or equal to 1. Pythagorean Fuzzy Set (PFS) is focused in a new source to deal with uncertainty considering the membership sign "μ" and non-membership sign "ν" satisfying the conditions ($\mu + \nu \leq 1$) or ($\mu + \nu \geq 1$), and also, it continues that ($\mu 2 + \nu 2 + \pi 2$) = 1, where "π" is the Pythagorean fuzzy set index. Score function for the ranking level of Interval Valued Pythagorean Fuzzy Sets pointed out by Garg which idea next level of IFS. S P Saranya and G Charles Rabinson developed about Pythagorean Decagonal Transportation Problems with IBFS and Optimal Solution. In Section 1, introduction based on all the new terminologies. In Section 2, the formulation of the PEGFTP is presented. In Section 3, discusses the procedures of the proposed algorithm. In Section 4, a numerical example is presented, lastly in Section 5, some concluding remarks are exhibited.

2. Preliminaries

2.1 Ƒuzzy Set:Ƒuzzy set Å in ℛeal is given to be a set {(x, μÅ(x)) (|x□R }, μÅ(x): X tends to [0,1] is mapping called the degree of membership function of the fuzzy set Å and μÅ(x) is called the membership value of x □X in the fuzzy set Å. These membership values are represented by real numbers lying in [0, 1].

2.2 Ƒuzzy Number: Fuzzy number is formulated as a fuzzy set defining a fuzzy interval in the real number. Normally a fuzzy numbers is represented by two extreme points. It is a fuzzy set the following conditions:

- Convex fuzzy set.
- Normalized fuzzy set.
- It should be a real number.
- It ís a membership function and also it is a piecewise continuous.

2.3 Intuitionistic Ƒuzzy Set: Let X is a Classical set, a Intuitionistic fuzzy set is an object having the form $\mathring{A}^{I}$ = {(x, (ẟI (x), σI (x))/x □ X},where the function ẟI (x): X→ [0, 1] and σI (x): X→[0, 1] are the degree of membership and non-membership of the element x □ X.

2.4 Pythagorean Ƒuzzy Set: Let X is a Classical set, a Pythagorean fuzzy set is an object having the form P = {(x, (ẟP (x), σP (x))/x □ X },where the function ẟP (x): X→ [0, 1] and σP (x): X→[0, 1] are the degree of membership and non-membership of the element x □ X to P, respectively. Also for every x□ X, it holds that 0 ≤ (ẟP (x)) 2 + (σP (x)) 2 ≤ 1. Suppose (ẟP (x))2 + (σP (x))2 ≤ 1 then there is a degree of indermînancy of x Xto A defined by $\pi P(x) = \sqrt{1-[\delta_p(x)^2+\sigma_p(x)^2]}$and πP(x) □[0,1]. It follows (δP (x))2 + (σP (x))2 $+\pi_p(x)^2$ =1. Otherwise πP(x) = 0. (δP (x))2 $+\sigma_p(x)^2 = 1$.

2.5Arithmetic Operations

Let $\alpha_1^p = (\delta_i^p, \sigma_u^p)$ and $\beta_1^p = (\delta_j^p, \sigma_v^p)$ be two Pythagorean Fuzzy Numbers (PFNs). Then the arithmetic operations are as follows:

(i) Additive property: α_1^p□β_1^p= (√(δ_i^p)2+ (δ_j^p)2−(δ_i^p)2 (δ_j^p))2,$\sigma_u^p.\sigma_v^p$)

(ii) Multiplicative property: α_1^p□β_1^p = (δ_i^p) .δ_s^p √(σ_u^p)2+ (σ_v^p)2−(σ_u^p))2 (σ_v^p)2)

(iii) Scalar product: kα_1^p = (√1− (1−δ_i^p),(σ_u^p)□)

where k is nonnegative constant .i.e.k > 0

2.6 Comparison of two PFNs

Let $\alpha_1^p = (\delta_i^p, \sigma_u^p)$ and $\beta_1^p = (\delta_j^p, \sigma_v^p)$ be two Pythagorean Fuzzy Numbers such that the score and accuracy function are as follows:

(i) Score function: S(α_1^p) = $\frac{1}{2}$(1−(δ_i^p)2−(σ_u^p)2)

(ii) Accuracy function: (α_1^p) = (δ_i^p)2+(σ_u^p)2

Then the following five cases arise:

Case 1: If $\alpha_1^p > \beta_1^p$ iff S(α_1^p)> S(β_1^p)

Case 2: If $\alpha_1^p < \beta_1^p$ iff S(α_1^p)< S(β_1^p)

Case 3: If S(α_1^p)= S(β_1^p) and H(α_1^p)< H(β_1^p) ,then $\alpha_1^p < \beta_1^p$

Case 4: If S(α_1^p)= S(β_1^p) and H(α_1^p)>H(β_1^p) ,then $\alpha_1^p > \beta_1^p$

Case 5: If S(α_1^p)= S(β_1^p) and H(α_1^p)= H(β_1^p) ,then $\alpha_1^p = \beta_1^p$.

2.7 Ranking Technique

Let P = {(x , (δP (x), σP (x))} a Pythagorean fuzzy number. The ranking R of ̃ on the set of Pythagorean fuzzy number is defined as follows:

R (P) = {(δP (x))2 + (σP (x))2}/2

2.8Mathematical Formulation of Pythagorean fuzzy transportation Problem

The Pythagorean fuzzy transportation problem can be represented in the form of n x n cost table $[C_{ij}]$after defuzzication as given below.

The costs $[C_{ij}]$ = (δP (x), σP (x)) are Pythagorean fuzzy numbers. The goal is to minimize the Pythagorean fuzzy cost incurred in transportation effectively.

The Pythagorean fuzzy transportation problem can be mathematically expressed as

Min Z = $\sum_{i=1}^{m}\sum_{j=1}^{n} C_{p_{ij}} X_{p_{ij}}$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = a_{p_i}$, i = 1,2,……,m.

$\sum_{i=1}^{m} x_{ij} = b_{p_j}$, j = 1,2,……,n.

$x_{ij} \geq$ 0, for all i and j.

If the total Pythagorean fuzzy supply from all sources equal to the total Pythagorean fuzzy demand in all destination then the Pythagorean fuzzy transportation problem is said to be balanced.

$\sum_{j=1}^{n} a_{p_i} = \sum_{j=1}^{n} b_{p_j}$, otherwise it is called unbalanced.

3. Pythagorean Enneagonal Fuzzy Numbers (PEGFN)

A Pythagorean Enneagonal fuzzy number $\left(a_p^1, a_p^2, a_p^3, a_p^4, a_p^5, a_p^6, a_p^7, a_p^8, a_p^9\right)$ can be defined as and the membership function is defined as

$$\mu_{A_p}(x) = \begin{cases} 0, & x \le a_1^p \\ \frac{(x - a_1^p)}{(a_2^p - a_1^p)}, & a_1^p \le x \le a_2^p \\ \frac{(x - a_2^p)}{(a_3^p - a_2^p)}, & a_2^p \le x \le a_3^p \\ \frac{(x - a_3^p)}{(a_4^p - a_3^p)}, & a_3^p \le x \le a_4^p \\ \frac{(x - a_4^p)}{(a_5^p - a_4^p)}, & a_4^p \le x \le a_5^p \\ 1 & , x = a_5^p \\ \frac{(a_5^p - x)}{(a_6^p - a_5^p)}, & a_5^p \le x \le a_6^p \\ \frac{(a_6^p - x)}{(a_7^p - a_6^p)}, & a_6^p \le x \le a_7^p \\ \frac{(a_7^p - x)}{(a_8^p - a_7^p)}, & a_7^p \le x \le a_8^p \\ \frac{(a_8^p - x)}{(a_9^p - a_8^p)}, & a_8^p \le x \le a_9^p \\ 0, & otherwise \end{cases}$$

Enneagonal Pythagorean Ƒuzzy Number

Let $\alpha_1^p = (\delta_1^p, \sigma_1^p)$, $\alpha_2^p = (\delta_2^p, \sigma_2^p)$, $\alpha_3^p = (\delta_3^p, \sigma_3^p)$, $\alpha_4^p = (\delta_4^p, \sigma_4^p)$, $\alpha_5^p = (\delta_5^p, \sigma_5^p)$, $\alpha_6^p = (\delta_6^p, \sigma_6^p)$, $\alpha_7^p = (\delta_7^p, \sigma_7^p)$, $\alpha_8^p = (\delta_8^p, \sigma_8^p)$, $\alpha_9^p = (\delta_9^p, \sigma_9^p)$ are theEnneagonal Pythagorean Fuzzy Numbers (HEGPFNs).

Proposed Approach of Pythagorean Enneagonal Fuzzy Transportation Problem

Step 1: Test whether the given Pythagorean Enneagonal fuzzy transportation problem is balance or not.

(i) If PEFTP is a balanced (i.e., the total Pythagorean supply is equal to the total Pythagorean demand) then go to step 3.

(ii) If it is an unbalanced (i.e., the total Pythagorean supply is not equal to the total Pythagorean demand) then go to step 2.

Step 2: Introduce dummy rows and /or dummy columns with zero Pythagorean fuzzy costs (enneagonal number)to convert a balanced PEFTP.

Step 3: Using the ranking function as mentioned find the rank of each cell C_{ij} of the chosen Pythagorean fuzzy cost matrix.

Step 4: If demand is greater than supply then the respective cell cost will be zeroby Zero Cost Method. After complete this process then apply VAM method.

Step 5: Proceed by the VAM method to find the initial basic feasible solution and if m+n-1 = No. of allocations, then apply MODI method to obtain the optimal solution.

4. Numerical Examples

The input data taken from GJC Pvt.Ltd Trichy. Enneagonal Pythagorean fuzzy transportation problem is given bellow. The optimal aim of the process is to minimize the transportation cost and maximize the profit.

Pythagorean Fuzzy Transportation Problem with Enneagonal Numbers

	MACH	THUR	ARI	P Supply
LAL	M(0.7,0.3,0.6,0.2,0.5, 0.8,0.4,0.7,0.4) NM(0.5,0.4,0.1,0.9, 0.3,0.1,0.9,0.6,0.8)	M(0.4,0.9,0.1,0.3,0.5, 0.2,0.1,0.4,0.8) NM(0.1,0.2,0.7,0.5, 0.7,0.6,0.9,0.6,0.1)	M(0.6,0.4,0.8,0.7,0.5, 0.6,0.9,0.4,0.8) NM(0.1,0.6,0.4,0.3, 0.1,0.3,0.2,0.6,0.1)	(3,6,9, 12,15, 18,21, 24,27)
PAL	M(0.3,0.4,0.7,0.9,0.5, 0.6,0.8,0.4,0.8) NM(0.7,0.6,0.4,0.1, 0.1,0.3,0.2,0.6,0.1)	M(0.6,0.4,0.2,0.7,0.5, 0.3,0.6,0.4,0.8) NM(0.1,0.6,0.9,0.3, 0.1,0.7,0.2,0.6,0.1)	M(0.4,0.3,0.6,0.8,0.5, 0.2,0.4,0.7,0.8) NM(0.5,0.4,0.1,0.5, 0.3,0.7,0.9,0.6,0.1)	(5,6,7, 8,9,10, 11,12, 13)
SAM	M(0.7,0.3,0.1,0.4,0.5, 0.2,0.1,0.4,0.8) NM(0.1,0.7,0.7,0.6, 0.7,0.6,0.9,0.6,0.1)	M(0.6,0.4,0.7,0.1,0.5, 0.6,0.2,0.4,0.8) NM(0.3,0.6,0.4,0.8, 0.1,0.3,0.2,0.6,0.1)	M(0.1,0.3,0.6,0.7,0.5, 0.8,0.2,0.7,0.3) NM(0.5,0.4,0.1,0.2, 0.3,0.1,0.9,0.5,0.6)	(5,7,9, 11,13, 15,17, 19,21)
P Demand	(1,4,7, 10,13,16, 19,22, 25)	(2,4,6, 8,10,12, 14,16,18)	(10,11,12, 13,14,15, 16,17,18)	

Solution:

Step 1: The Enneagonal Pythagorean fuzzy numbers transportation problem is converted into Enneagonal fuzzy transportation problem using the ranking method. Using Ranking Method

	MACH	THUR	ARI	P Supply
LAL	(0.37,0.13,0.19,0.43, 0.17,0.33,0.49,0.43, 0.40)	(0.09,0.43,0.25,0.17, 0.37,0.20,0.41,0.26, 0.33)	(0.19,0.26,0.40,0.29,0 .13, 0.23,0.43,0.26,0.33)	(3,6,9, 12,15, 18,21, 24,27)
PAL	(0.29,0.26,0.33,0.41, 0.13, 0.23,0.34,0.26,0.33)	(0.19,0.26,0.43,0.29, 0.13, 0.29,0.20,0.26,0.33)	(0.21,0.13,0.19,0.45,0 .17, 0.27,0.49,0.43,0.33)	(5,6,7, 8,9,10, 11,12, 13)
SAM	(0.25,0.29,0.25,0.26, 0.37, 0.20,0.41,0.26,0.33)	(0.23,0.26,0.33,0.33, 0.13, 0.23,0.04,0.26,0.33)	(0.13,0.13,0.19,0.27,0 .17, 0.33,0.43,0.37,0.23)	(5,7,9, 11,13, 15,17, 19,21)
P Demand	(1,4,7, 10,13,16, 19,22, 25)	(2,4,6, 8,10,12, 14,16,18)	(10,11,12, 13,14,15, 16,17,18)	

Step 2: Arrange the numbers in ascending order in each cells

	MACH	THUR	ARI	P Supply
LAL	(0.13, 0.17,0.19, ,0.33,0.37,0.40, 0.43, 0.43, 0.49)	(0.09, 0.17, 0.20, 0.25, 0.26 ,0.33, 0.37, 0.41 0.43)	(0.13,0.19, 0.23, 0.26, 0.26,0.29, 0.33,0.40,0.43)	**(**3,6,9, 12,15, 18, 21, 24,27**)**
PAL	(0.13,0.23,0.26, 0.26,0.29,0.33, 0.33, 0.34 0.41)	(0.13,0.19,0.20, 0.26,0.26,,0.29, 0.29,0.33, 0.43)	(0.13, 0.17,0.19, 0.21,0.27, 0.33 , 0.43,0.45, 0.49)	**(**5,6,7, 8,9,10, 11,12, 13**)**
SAM	(0.20,0.25,0.25, 0.26,0.26,0.29, 0.33,0.3,0.41,)	(0.04,0.13,0.23, 0.23,,0.26,0.26, 0.33, 0.33,0.33,)	(0.13,0.13, 0.17, 0.19, 0.23,0.27, 0.33, 0.37,0.43)	**(**5,7,9, 11,13, 15,17, 19,21**)**
P Demand	**(**1,4,7, 10,13, 16, 19,22, 25**)**	**(**2,4,6, 8,10, 12, 14,16,18**)**	**(**10,11,12, 13, 14,15, 16,17,18**)**	

Step 3: Find the difference between Maximum and Minimum for each cells

	MACH	THUR	ARI	P Supply
LAL	0.36	0.40	.030	24
PAL	0.28	0.30	0.36	8
SAM	0.21	0.29	0.30	16
P Demand	24	16	8	

It is a balanced transportation problem since the total supply and the total demand is 48.

IBFS = (8 × 0.36) + (16 × 0.21) + (8 × 0.40) + (8 × 0.30) + (8 × 0.30)
= 14.24.

After testing Optimality all $\Delta_{ij} \geq 0$ so we got optimal solution.

5. Comparison Table

Sr. No.	NWCM	LCM	VAM	Optimal Solution	**Proposed Method**	**Optimal Solution**
1	15.76	14.40	14.24	14.24	**4.72**	**4.72**

6. Conclusion

This research paper has proposed a new ranking for Pythagorean Enneagonal fuzzy numbers. The proposed ranking is applied to expound Pythagorean Enneagonal fuzzy transportation problem. Further, a numerical example has determined whose costs are taken as Pythagorean Enneagonal fuzzy numbers .The proficiency of the proposed technique is shown in the comparison table. As a future research, the proposed algorithm may be used to solve Pythagorean fuzzy Assignment problem (any number) and Pythagorean fuzzy interval valued fuzzy assignment and Pythagorean fuzzy transportation problems with any number.

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